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EFFECT OF THE DIRECTION AND MAGNITUDE OF AN EXTERNAL STRESS FIELD ON THE TRAJECTORY OF A STAR-SHAPED SYSTEM OF CRACKS

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Zone models [1-5] are usually used to describe fracture from explosions in solids. In the general case, three zones are distinguished in these models: the crushing zone, adjacent to the explosive charge; an intermediate zone of radial cracks; the elastic outer zone. In the formulation that will be used here, there is no plastic zone, i.e. the solution is valid for "weak" charges. Instead of two successive zones, we use the solution of the problem of the stress state of an elastic plane with a system of cracks propagating from the boundary of a cylindrical blast cavity loaded internally by an assigned load. In contrast to zone models — in which the effect of the explosion is initially presumed to be symmetric and the external stress field is characterized by a single quantity (counter pressure) — in our formulation we consider the external field created by the stresses p and q, acting at infinity in orthogonal directions, and the angle γ (between the Ox axis and the direction of the stress p). The method of solution we will use makes it possible to qualitatively evaluate the effect of changes in the parameters of the external field p, q, and γ on the trajectory of an initially ordered system of radial cracks, i.e. the deformation of the zone of radial cracks in the zone model used to describe an underground explosion. We will use a quasistatic approximation. We examine two variants of crack-loading: when the detonation products penetrate the cracks; when they do not penetrate the cracks. We also evaluate the effect of tamping and the presence of an adjacent well charge on the main parameters characterizing the growth of the star-shaped system of cracks. The present study is a logical extension of [5-7], where the authors made use of an algorithm for solving singular integral equations [8-11].

Quasistatic Formulation of the Problem. We will examine an isotropic elastic state containing N smooth curvilinear slits that begin at the boundary of a circular hole. We assume that uniform stresses of intensities p and q act at infinity in two mutually orthogonal directions. Each slit L_k is referred to its own local coordinate system $x_kO_ky_k$, $k = \overline{1, N}$ (Fig. 1). The relationship between the points of the plane in the main and local coordinate systems is determined by the expression

$$z = x + iy = z_k e^{i\alpha_k} + z_k^0 = (x_k + iy_k)e^{i\alpha_k} + x_k^0 + iy_k^0,$$

where α_k is the angle between the axes Ox and O_kx_k; (x^0_k, y^0_k) is the origin of the coordinates of the k-th local system in the main system.

We assume that the shape of each slit L_k in its local system is known and is given by the parametric equation

$$t_k = \omega_k(\xi) = x_k(\xi) + iy_k(\xi), \ |\xi| \leqslant 1, \ t_k \in L_k. \tag{1}$$

Then the solution of the problem of the theory of elasticity for the case when the normal and shear stresses are assigned on the contours of the slits

$$N_k^{\pm} + iT_k^{\pm} = p_k^0(t_k), \ t_k \in L_k, \ k = \overline{1, N} \tag{2}$$

(the plus sign pertaining to the top edge of the slit, the minus to the bottom edge) reduces to finding the solution $g_k'(\xi)$ ($k = \overline{1, N}$) of a system of N complex singular integral equations of the form [10]

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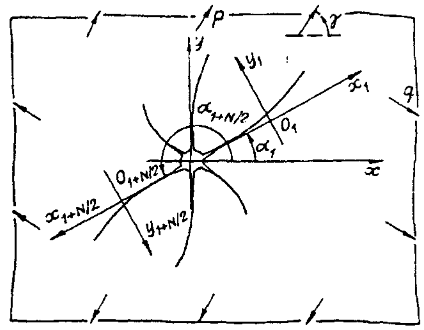

Fig. 1

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{-1}^{1}[R_{kn}(\xi,\eta)g_k'(\xi) + S_{kn}(\xi,\eta)\overline{g_k'(\xi)}]d\xi = P_n(\eta), \qquad |\eta| \leqslant 1,\ n = \overline{1,N}. \tag{3}$$

Here, $g_k'(\xi) = g_k'(t_k)\omega_k(\xi)$; $g_k'(t_k)$ is the derivative of the displacement discontinuity:

$$\frac{d}{dt_k}[(u_k + iv_k)^+ - (u_k + iv_k)^-] = \frac{i(1+\varkappa)}{2\mu}g_k'(t_k) \ (t_k \in L_k);$$

$\varkappa = 3 - 4\nu$; ν is the Poisson's ratio (plane strain); σ_0 is the pressure acting in the hole; μ is the shear modulus;

$$R_{kn}(\xi,\eta) = R_{kn}(T_k, T_n) = e^{i\alpha_k}(\zeta_{kn}(\xi,\eta) + \rho_n r_{kn}(\xi,\eta));$$

$$S_{kn}(\xi,\eta) = S_{kn}(T_k, T_n) = e^{-i\alpha_k}(\overline{\zeta_{kn}(\xi,\eta)} + \rho_n s_{kn}(\xi,\eta));$$

$$\zeta_{kn}(\xi,\eta) = \frac{1}{a_2} + \frac{1}{T_n\bar{a}_1} + \frac{A}{T_k a_1^2} + \frac{1}{T_n};$$

$$r_{kn}(\xi,\eta) = -\frac{2\bar{T}_n A}{a_1^3} + \frac{1}{\bar{a}_2} + \frac{1}{\bar{T}_n a_1} + \frac{A(3\bar{T}_n T_k - 1)}{T_k\bar{T}_n^2 a_1^3} + \frac{1}{\bar{T}_n};$$

$$s_{kn}(\xi,\eta) = -\frac{a_2}{\bar{a}_2^2} + \frac{1}{\bar{T}_k\bar{T}_n^2} + \frac{3\bar{T}_n T_k - 2}{\bar{T}_n^3 a_1^2} - \frac{T_n(2\bar{T}_n T_k - 1)}{\bar{T}_n^2 a_1^2} + \frac{2 - T_n\bar{T}_k}{\bar{T}_n^3};$$

$$A = 1 - T_k\bar{T}_k;\ a_1 = \bar{T}_n T_k - 1;\ a_2 = T_k - T_n;$$

$$\rho_n = \frac{\overline{\omega_n'(\eta)}}{\omega_n'(\eta)}e^{-2i\alpha_n};$$

$$P_n(\eta) = -p_n^0(\eta) - \Bigg\{p_+ - p_- 2\mathrm{Re}(e^{2i\gamma}/T_n^2) + \rho_n\left[p_-\left(\frac{2e^{-2i\gamma}T_n}{\bar{T}_n^3} - e^{2i\gamma} - \frac{3e^{-2i\gamma}}{\bar{T}_n^4}\right) + \frac{p_+ + \sigma_0}{\bar{T}_n^2}\right]\Bigg\};$$

$$p_+ = (p+q)/2;\ p_- = (p-q)/2;$$

$$T_k = T_k(\xi) = e^{i\alpha_k}\omega_k(\xi) + z_k^0;\ T_n = T_n(\eta) = e^{i\alpha_n}\omega_n(\eta) + z_n^0$$

$\omega_k(\xi)$ and $\omega_n(\eta)$ are given by the Eqs. (1).

The resulting kernels of integral equations (3) have the properties $R_{kn}(-1,\eta) = S_{kn}(-1,\eta) = 0$, since the cracks begin from the boundary of a circular hole in the given problem [10, 11].

We will assume that N is an even number. This restriction is of a purely procedural nature, since the problem has central symmetry with even N, and we can use this fact to reduce the order of the system of equations that is to be solved. In fact, by virtue of the central symmetry we have

$$g_i'(\xi) = g_{i+N/2}'(\xi),\ T_i(\xi) = -T_{i+N/2}(\xi),$$
$$\alpha_{i+N/2} = \pi + \alpha_i,\ i = \overline{1, N/2},$$

which makes it possible to change over to a system of N/2 integral equations of type (3) having the kernels

$$R_{kn}^{*}(\xi, \eta) = R_{kn}(T_k, T_n) - R_{kn}(-T_k, T_n),$$
$$S_{kn}^{*}(\xi, \eta) = S_{kn}(T_k, T_n) - S_{kn}(-T_k, T_n),\ k, n = \overline{1, N/2}. \tag{4}$$

The solution of the equations is sought in the form

$$g_k'(\xi) = \varphi_k(\xi)/\sqrt{1 - \xi^2}. \tag{5}$$

Using Gauss' formulas of integration [10] and (3) with allowance for (4-5), we obtain a system of $n\left(\frac{N}{2} - 1\right)$ linear complex algebraic equations:

$$\sum_{k=1}^{N/2}\sum_{i=1}^{N/2} [R_{kj}^{*}(\xi_i, \eta_m)\varphi_k(\xi_i) + S_{kj}^{*}(\xi_i, \eta_m)\overline{\varphi_k(\xi_i)}] = 2nP_j(\eta_m),\quad j = \overline{1, N/2},\ m = \overline{1, n-1}. \tag{6}$$

Here, n determines the order of approximation of the solution, while

$$\xi_i = \cos\frac{\pi(2i-1)}{2n},\ i = \overline{1, n},\ \eta_m = \cos\frac{\pi m}{n}\ (m = \overline{1, n-1})$$

are zeros of Chebyshev polynomials $T_n(\xi) = \cos(n \arccos\xi)$, $U_{n-1}(\eta) = \sin(n \arccos\eta)/\sqrt{1 - \eta^2}$ of the first and second kinds, respectively. We adopt the following [10] as the auxiliary conditions that close system (6)

$$\varphi_k(-1) = 0,\ k = \overline{1, N/2}, \tag{7}$$

which ensures that the displacements are finite on the left end of slits extending to the boundary of the hole.

The main characteristics used in crack theory — the stress-intensity factors for the singularity $(2r)^{-1/2}$, $r/l << 1$ — are determined by solving system (6-7) with the formula [10]

$$k_{1k}^{\pm} - ik_{2k}^{\pm} = \mp\sqrt{|\omega_k'(\pm 1)|}\,\frac{\varphi_k(\pm 1)}{\omega_k'(\pm 1)}, \tag{8}$$

where

$$\varphi_k(1) = -\frac{1}{n}\sum_{i=1}^{n}(-1)^i\varphi_k(\xi_i)\operatorname{ctg}\frac{\pi(2i-1)}{4n};$$
$$\varphi_k(-1) = \frac{1}{n}\sum_{i=1}^{n}(-1)^{i+n}\varphi_k(\xi_i)\operatorname{tg}\frac{\pi(2i-1)}{4n}\ (k = \overline{1, N/2}).$$

The fracture criterion σ_ϑ [8] was used to construct the trajectory of the cracks. In accordance with this criterion, any crack will grow in the direction in which the normal stress is maximal. This direction is determined by the angle ϑ_*, which is described from the positive direction by a tangent drawn to the tip of the crack and is given by the expression

$$\vartheta_* = 2\operatorname{arc\,tg}\frac{k_1 - \sqrt{k_1^2 + 8k_2^2}}{4k_2}. \tag{9}$$

At $k_1 \le 0$, we will assume that the crack is stopped — its edges are superimposed. The limit equilibrium equation is obtained from the condition that the coefficient with the singularity $(2r)^{-1/2}$ in the expansion for $\sigma_{\vartheta\vartheta}$ near the crack tip be equal to the critical value of the stress-intensity factor K_{Ic} when $\vartheta = \vartheta_*$ [8]:

$$\frac{1}{4}\cos^3\left(\frac{\vartheta_*}{2}\right)\ [k_1 + 3\sqrt{k_1^2 + 8k_2^2}\ ] = K_{Ic}/\sqrt{\pi}. \tag{10}$$

An algorithm used to construct the trajectory step by step was described in detail in [6, 7, 10].

In calculating the growth of a system of cracks originating from a hole, we used the Jones–Miller adiabatic curve [12] for a cylindrical charge of trotyl:

$$\sigma_0(r) = p_{00}\begin{cases}(r/r_0)^{-2\gamma_1}, & r \le r_*,\\ (r_*/r_0)^{-2\gamma_1}(r/r_*)^{-2\gamma_2}, & r > r_*.\end{cases} \tag{11}$$

Here, $p_{00} = 10^{10}$ Pa; $\gamma_1 = 3$; $\gamma_2 = 1.27$; $r_* = 1.89r_0$; r_0 is the radius of the explosive charge; r is the "corrected" radius of the cylindrical cavity occupied by the detonation products at a given moment in the growth of the system of cracks.

Main Propositions and Simplifications. We will examine two limiting cases: a stationary tamping; the absence of a tamping. In the second variant, allowance is made for the escape of gases in a quasisteady approximation. The equation of state has the form

$$p = A\rho^\gamma, \tag{12}$$

where the choice of γ agrees with (11). The Bernoulli equation

$$\frac{v_i^2}{2} + \frac{c_i^2}{\gamma - 1} = \frac{c_{i-1}^2}{\gamma - 1} = \frac{\gamma}{\gamma - 1}\frac{p_{i-1}}{\rho_{i-1}}.$$

is satisfied for any i-th stage of crack growth. We assume that v^*_i — the limiting rate of flow of gases out of the mouth of the well — is equal to c_i. Then

$$v_i^* = c_{i-1}\sqrt{\frac{2}{\gamma + 1}} \tag{13}$$

and gas discharge per unit time

$$Q_i = \rho_i v_i^* \pi R^2 \tag{14}$$

(R is the radius of the well). Taking (12) into account, we find from [13] that

$$v_i^* = c_i,\ c_i^2 = \frac{\gamma p_i}{\rho_i} = \frac{2}{\gamma + 1}\frac{\gamma p_{i-1}}{\rho_{i-1}},\ \frac{p_i}{p_{i-1}} = \frac{\rho_i}{\rho_{i-1}}\frac{2}{\gamma + 1} = \left(\frac{\rho_i}{\rho_{i-1}}\right)^\gamma.$$

We use the last equality to obtain

$$\rho_i = \rho_{i-1}\left(\frac{2}{\gamma + 1}\right)^{1/(\gamma-1)},\quad p_i = p_{i-1}\left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)} \tag{15}$$

Using (13) and (15), we write the expression for the discharge (14) in the form

$$Q_i = \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)} \sqrt{\gamma p_{i-1}\rho_{i-1}}\ \pi R^2. \tag{16}$$

We examined two variants of loading of the crack system: 1) gas pressure concentrated in the blast cavity and gases not entering the cracks, i.e. the cracks are free of stresses (in (2), $p_{k0} = 0$, $k = \overline{1, N}$); 2) the gaseous detonation products enter the cracks. In this case, we assume that there is sufficient time for gas to fill the entire length of the crack as the latter advances. Meanwhile, at each stage of its propagation, pressure in the blast cavity remains the same as the pressure in the cracks, i.e. $\sigma_0 = p_k^0$, $k = \overline{1, N}$. We further assume that gas pressure obeys adiabatic law (11-12) as the system of cracks grows.

If we consider the completely symmetric case ($p = q = q_0$) for initially straight cracks propagating from the boundary of a circular hole, the problem reduces to the solution of a single integral equation. Calculations we performed showed that the asymptotic formulas proposed in [13-15] for the stress-intensity factors

$$
\begin{aligned}
k_1 &= \frac{\sigma_0 R}{\sqrt{N(L+1)R}} + 2q_0\sqrt{(L+1)R}\,\frac{\sqrt{N-1}}{N}, \quad p^0 = 0, \\
k_1 &= 2(p^0 + q_0)\sqrt{(L+1)R}\,\frac{\sqrt{N-1}}{N}, \quad \sigma_0 = p^0
\end{aligned}
\tag{17}
$$

have a relative error not exceeding 0.5% for $N \geq 4$ and $L \geq 2$. In the second case, when $\sigma_0 = p^0$, approximate formulas were obtained for the volume of the cracks present per unit length of the charge:

$$
\begin{aligned}
V &= \pi R^2 \frac{4(1-\nu^2)}{E}(p^0 + q_0)L(b_1 + b_2 L) = \pi R^2 A_0 (p^0 + q_0) f(L), \\
A_0 &= \frac{4(1-\nu^2)}{E}.
\end{aligned}
\tag{18}
$$

Here, E is the elastic modulus; $L = l/R$ is the dimensionless length of the cracks; the coefficients b_1 and b_2 depend on the number of cracks:

N	2	4	6	8	10	20
b_1	0.260	0.572	0.700	0.769	0.817	0.930
b_2	0.516	0.766	0.850	0.895	0.922	0.982

The relative error of these formulas for $L > 3$ is no greater than 3%. In the general case, when $p \neq q$, we can replace $(p^0 + q_0)$ in (18) by $(p^0 + p_{0i})$ for the i-th crack, where

$$
P_{0i} = p_+ - p_-\cos(2(\gamma - \alpha_i)),
$$

while α_i is the angle of inclination of a chord connecting the beginning and end of the i-th crack to the Ox axis.

We assume that the gases do not escape from the well until straight cracks of the length $l_0 = 2R$ are formed. The adiabatic curve (11) is used to find the pressure p and density ρ at $r = R$ for the moment when the gases fill the well. Well radius assumes the value R_1 under the influence of the pressure in the well and the external stress field ($p < 0$, $q < 0$ denotes compression). This quantity can be approximately determined from the expression

$$
R_1 = R\left[1 + \frac{1+\nu}{E}(p_1 + 2p_+(1-\nu))\right]
$$

($p_1 = p\xi$ is the pressure corresponding to this radius). Then taking (11) into account, we obtain the equation

$$
\frac{R_1}{R} = \left(\frac{p}{p_1}\right)^{1/2\gamma}, \quad 1 + \frac{1+\nu}{E}[p\xi + 2p_+(1-\nu)] = \xi^{-1/2\gamma},
$$

Having solved this, we find that $p_1 = p\xi$ and $\rho_1 = \rho\xi^{1/\gamma}$. If the detonation products do not enter the cracks, then the initial values of pressure p_2 and density ρ_2 used to calculate the first stage of crack growth are found from the equation

$$
\frac{R_2}{R_1} = \xi^{-1/2\gamma}, \quad p_2 = p_1\xi, \quad \rho_2 = \rho_1\xi^{1/\gamma}, \quad \bar{l}_0 = l_0/R
$$

$$
\left(R_2 = R\left[1 + \frac{1+\nu}{E}p_1\xi(1 + (1-\nu)\ln(1+\bar{l}_0)) + 2p_+(1+\bar{l}_0)\frac{1-\nu^2}{E}\right]\right).
$$

The expression for R_2 was established on the basis of the "columnar elasticity" assumption used in the zone model that describes underground explosions.

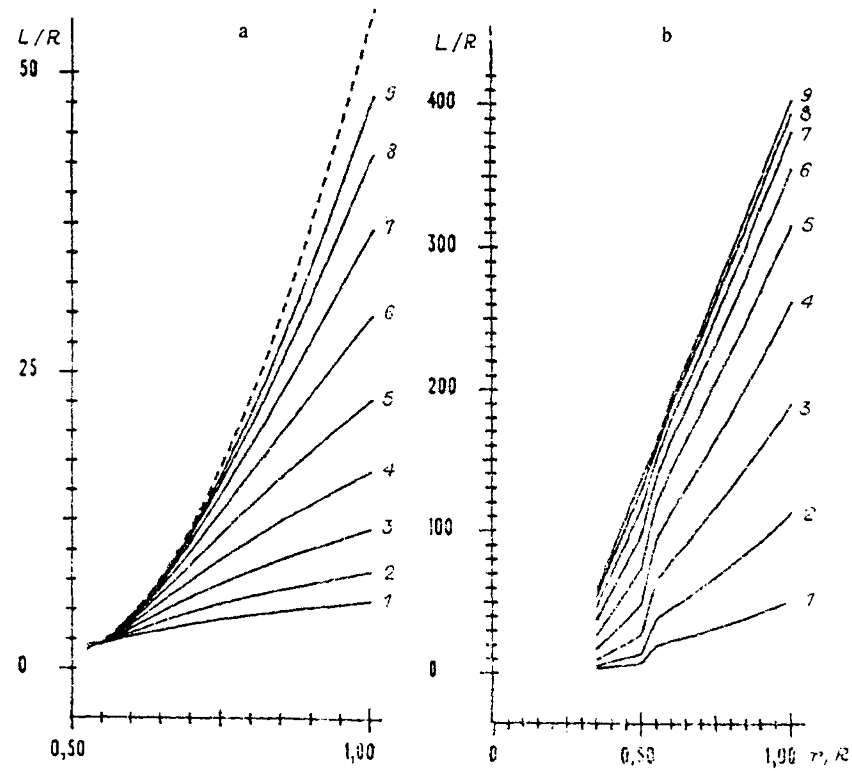

Fig. 2

We introduce the time-step parameter $\Delta t = 2R/kv_0$ (k is a positive number and v_0 is the limiting rate of crack propagation in the given medium). We proceed as follows in allowing for changes in gas pressure and density as crack growth occurs. Let gas pressure and density in the well at the i-th step be known. Then their values for calculation of the step (i + 1) are found on the basis of engineering considerations. We write the equation for the loss of mass by the gas during the time Δt with allowance for (16)

$$\Delta M = L_0(\Delta V\rho_i + V_i\Delta\rho) = -B\pi R^2\sqrt{\gamma p_i\rho_i}\,\Delta t,$$

where L_0 is the length of the explosive charge; BB; $B = \left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/2(\gamma-1)}$;

$$V_i = \pi R^2\bar{R}_i^2 = \pi R^2\left[1 + \frac{1+\nu}{E}p_i(1 + (1-\nu)\ln(1+\bar{l}_i)) + 2p_+(1+\bar{l}_i)\frac{1-\nu^2}{E}\right]^2.$$

Assuming that $p_{i+1} = p_i(1 - \varepsilon)$ $(\varepsilon > 0,\ \varepsilon \ll 1)$, we use this equation to obtain

$$\varepsilon = \left[\frac{2(1-\nu^2)}{E}\left(\frac{p}{1+\bar{l}_i} + 2p_+\right)\Delta\bar{l}_i + \frac{B\Delta t}{L_0\bar{R}_i}\sqrt{\frac{\gamma p_i}{\rho_i}}\right]\Big/\left[\frac{\bar{R}_i}{\gamma} + 2p_i\frac{1+\nu}{E}(1 + (1-\nu)\ln(1+\bar{l}_i))\right]$$

while the density for the next step $\rho_{i+1} = \rho_i(1 - \varepsilon/\gamma)$.

The following algorithm is used in the second limiting case, when the gas completely fills the system of cracks and the pressure in them equals gas pressure in the blast cavity. We assume that the i-th state p_i, ρ_i, $\bar{l}_{i,k}$ is known and that we also know the lengths $\bar{l}_{i+1,k}$ $(k = \overline{1, N/2})$. We again use the equation for the mass lost by the gas during Δt. Here, the volume per unit length

$$V_i = \pi R^2\left[1 + A_{00}\sum_{k=1}^{N/2} f(\bar{l}_{i,k})\,(p_i + P_{0k})\right] = \pi R^2\bar{V}_i,\ A_{00} = \frac{2A_0}{N}.$$

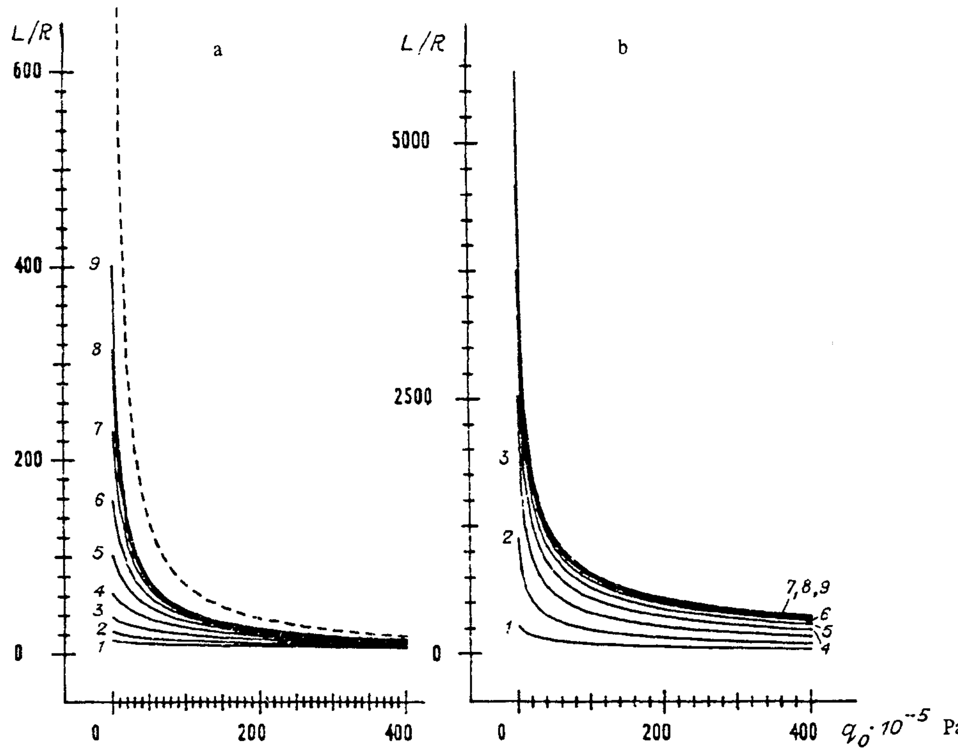

Fig. 3

Assuming that $p_{i+1} = p_i(1 - \varepsilon)$ и $\rho_{i+1} = \rho_i\left(1 - \frac{\varepsilon}{\gamma}\right)$ $(\varepsilon > 0,\ \varepsilon << 1)$, we have

$$\varepsilon = \left[A_{00}\sum_{k=1}^{N/2} f(\bar{l}_{i+1,k})\,(p_i + P_{0k+1}) - A_{00}\sum_{k=1}^{N/2} f(\bar{l}_{i,k})\,(p_i + P_{0k}) + \frac{B\Delta t}{L_0}\sqrt{\frac{\gamma p_i}{\rho_i}}\right] \Bigg/ \left[\frac{V_i}{\gamma} + A_{00}p_i\sum_{k=1}^{N/2} f(\bar{l}_{i+1,k})\right],$$

thus determining gas pressure and density for the next stage.

As a result, a static problem is solved for each stage of crack growth, the solution being used to find the stress-intensity factors $k_1^{(i)}$ and $k_2^{(i)}$ $(i = \overline{1,\ N/2})$ (8) at the tip of each crack. We then use (9) and (10) for each crack to determine the corresponding coefficients $k_p^{(i)}$. Using the formula proposed in [16],

$$v_i = \begin{cases} v_0(1 - e^{-\beta}), & \beta > 0, \\ 0, & \beta \leq 0, \end{cases} \quad \beta = \frac{k_p^{(i)}}{K_{Ic}} - 1,$$

we find the velocity of each crack at the given moment of time and determine the increment δ_i to calculate its trajectory in the form

$$\delta_i = \frac{v_i\Delta t}{R} = \frac{2v_i}{kv_0},\ i = \overline{1, N/2}.$$

Formulas (18) for crack volume were obtained on the assumption that the cracks are straight and of the same length. As crack length in our calculations, we took the length of the chord connecting the beginning and end of the crack. Also, if some crack in the system was stopped, the calculation was continued with the initially chosen value of N, i.e. the coefficients b_1 and b_2 did not change. In the present quasistatic formulation, we take into account the change in the stress field at each stage of crack growth due to lengthening and curvature of the cracks, and we ignore the effect of dynamic factors. Naturally, given these assumptions, the calculated results will most likely be of a qualitative nature.

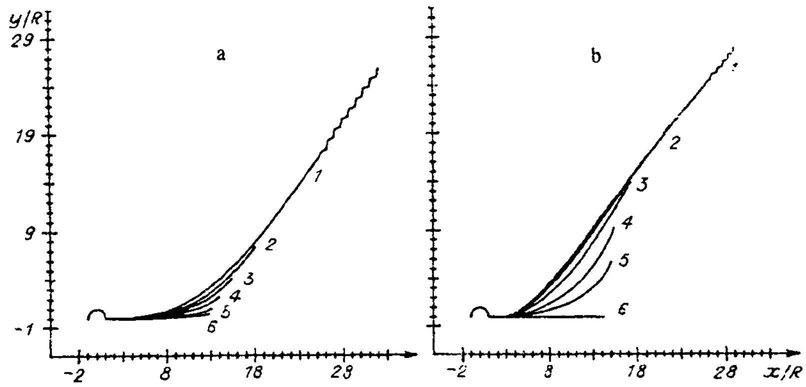

Fig. 4

Results of Numerical Calculations and Their Analysis. We performed numerical calculations with the use of data characteristic of sandstone and the following initial parameter values: $\sigma_c = 10^8$ Pa — compression limit; $E = 3\cdot10^{10}$ Pa; $\nu = 0.3$; $K_{Ic} = 3$ MPa·m$^{1/2}$; $R = 0.0525$ m; $v_0 = 650$ m/sec; $\rho_0 = 10^3$ kg/m^3 (the density of the explosive).

Analysis of Gas Discharge for the Final State of the Radial Crack Zone. As was noted above, the problem is simplified in the symmetric case ($p = q = q_0$): we obtain a single integral equation. The cracks will move in a straight line, and the algorithm for their propagation can be constructed using one parameter — the stress-intensity factor k_1. Nearly the same results were obtained from test calculations performed with the use of the integral equation and approximating formulas (17) for k_1. We therefore saved computing time by obtaining the results for the symmetric case from Eqs. (17). In these expressions, it was assumed that there was no tamping and that a symmetric system of ten straight cracks of length 2R was present at the initial moment of time. Figure 2 shows the dependence of the final size of the radial-crack zone L/R on the ratio of the radius of the charge r_0 to the radius of the well R with charge lengths $L_0 = 0.5$, 1, 2, 4, 8, 16, 32, 64, and 128 m (lines 1-9) and a counter pressure $q_0 = -35$ MPa (*a* shows data for the case when gases do not penetrate the cracks, while *b* shows data for the case when gases fill the cracks). Large values of L_0 naturally correspond to the lines that are higher in the figure. The dashed line corresponds to $L_0 = \infty$ — a stationary tamping. It should be noted that crack growth begins at $r_0/R > 0.5$ in Fig. 2a and at $r_0/R > 0.3$ in Fig. 2b. It can be seen from Fig. 2 that the absence of a tamping has a significant effect on the final size of the radial-crack zone for realistic charge lengths L_0 = 4-10 m (this is especially clear in Fig. 2a). Also, penetration of the cracks by gases increases the size of the fracture region by roughly one order. Figure 3 shows the effect of counter pressure q_0 and charge length L_0 on the final dimensions of the radial-crack zone at $r_0 = R$ (the notation corresponds to that in Fig. 2). The sharpest decrease in zone size occurs within the range of counter pressures up to 10 MPa.

Effect of Direction and Magnitude of the External Stress Field. Given sufficiently large q_0 and small L_0, the crack system is relatively small. Thus, assuming that appropriate initial parameters have been chosen, it is more realistic to calculate the trajectories of cracks in a nonuniform external field of compressive stresses ($p \neq q$) and to evaluate the effect of changes in its parameters on the trajectory of the cracks at the moment they stop. We performed several such calcualtions, and some of the results are discussed below.

Figure 4a, b, shows the trajectories with N = 2 — the right branches (the left branches are centrally symmetric relative to the center of the well). The calculations were performed for $\gamma = \pi/4$, $p = -55$ MPa, and different values of q. As above, variant *a* corresponds to the case when gases do not penetrate the cracks. Here, $L_0 = 4$ m, $r_0 = 0.045$ m, and q = 0.5, 15, 25, 35, 45, and 50 MPa (lines 1-6). The calculations in variant *b* were performed with $L_0 = 2$ m, $r_0 = 0.022$ m, and $-q$ = 15, 25, 35, 45, 50, and 55 MPa (lines 1-6). The smaller values of q/p correspond to longer cracks. At $q/p \to 1$, naturally, the trajectories tend toward linearity. For relatively small q/p, the cracks grow in the direction of the greatest compressive stress p. It should be noted that the qualitative reaction of the trajectories to a change in the external field — the ratio q/p — is the same in each case, although the calculations performed in variant *a* required a larger number of nodes n, i.e. the solution of a system of linear equations of greater dimensions. We therefore subsequently restricted ourselves to study of the variant in which gases fill the cracks.

Figure 5 shows the trajectories with N = 6 before the cracks stop. Figure 5a shows trajectories corresponding to the charge radii $r_0 = 0.018$, 0.02, and 0.022 m with $\gamma = 75°$, $L_0 = 2$ m, $p = -55$ MPa, and $q = -35$ MPa. We should note the general orientation of the longer cracks in the direction of the maximum compressive stress p; larger values of r_0 naturally

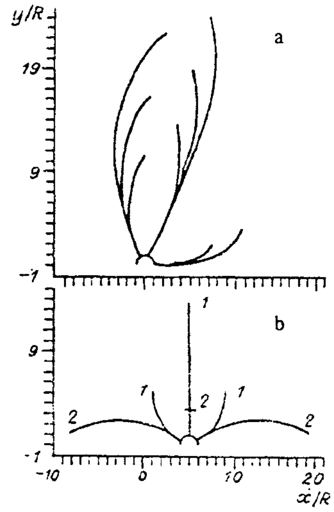

Fig. 5

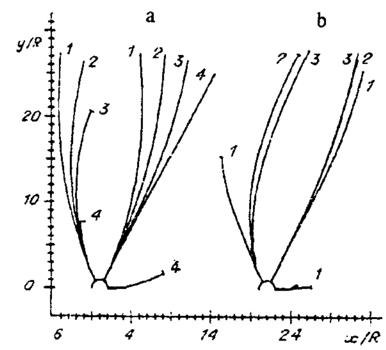

Fig. 6

correspond to larger cracks. Also, the cracks tend to straighten out as r_0 increases. Figure 5b shows trajectories calculated with $\gamma = \pi/2$, $L_0 = 2$ m, $r_0 = 2$ m, $r_0 = 0.018$ m, $p = -55$ MPa, $q = -35$ MPa (lines 1) and $p = -35$ MPa, $q = -55$ MPa (lines 2). Here, it is possible to clearly see that the trajectories of the growing cracks are oriented in the direction of the maximum compressive stress — p in the first case and q in the second case. At small r_0, the cracks that are nearly orthogonal to the direction of action of the maximum compressive stress undergo almost no growth.

We performed a large number of calculations for N = 2-10 in the case of a stationary tamping, which corresponds formally to $L_0 \to \infty$. Here, we changed the angle γ and the ratio q/p. The value of $r_0 = 0.0175$ m, a choice that allowed us to ignore the plastic zone — since pressure in the well was lower than σ_c at the moment it was filled with detonation products and some of the cracks had stopped. The results of calculations for N = 6 are shown to illustrate the typical pattern. Figure 6a shows the trajectoriess obtained with $p = -55$ MPa and $q = -45$ MPa. Lines 1-4 correspond to γ = 90, 80, 70, and 60°. The problem has symmetry relative to this direction at $\gamma = 60°$, so that the second crack moves in a straight line and the first and third cracks move along lines 4 — which are symmetric relative to this line. When $\gamma = \pi/2$, line 1 corresponds to the trajectories of the second and third cracks, which are symmetric relative to the vertical axis. The first crack stops in the variants being discussed, while the third crack stops in variants 3 and 4 (these positions are denoted by the lines drawn laterally). Figure 6b (at $\gamma = 70°$, $p = -55$ MPa) shows the effect of the orthogonal stress $-q$ = 50, 40, and 35 MPa (lines 1-3) on the trajectories of the cracks. There is almost no movement of the first crack, while the third crack travels a long distance until it stops with a reduced value of q/p. The lateral lines in Fig. 6 denote cracks that were stopped at the moment the calculation was ended.

An analysis of the results permits the following conclusions. While the problem is symmetric with p = q and cracks propagate in a straight line for any N, when $p \neq q$ (we assume that $|p| > |q|$) the cracks deviate from rectilinear growth simply as a result of the uniaxial stress field acting at infinity. This field has the intensity $\Delta p = p - q$ and the angle of inclination γ. Cracks in the star-shaped system tend to grow in the direction of the maximum compressive stress Δp. Cracks that are initially almost orthogonal to the direction of the stress field Δp are slowed the most or remain stationary. A change in Δp has less of an effect on the trajectory of a crack whose initial direction is close to the direction of the compressive field Δp. The remaining cracks deviate more sharply from a straight line with an increase in Δp and travel farther before stopping.

Effect of an Adjacent Well Charge. In symmetric problems concerning star-shaped cracks originating from one point [14] or propagating from the boundary of a circular hole [15], the expressions for the stress-intensity factors have the form $k_1 = 2p\sqrt{(L/N)}$, $k_1 = 2p[\sqrt{L}(N-1)/N]$, respectively. Here, p is the pressure acting in the cracks. This is reason to hope that the trajectories for the simpler problem of cracks growing from a single point will be close to the trajectories constructed above. In fact, test calculations performed for N = 6 with different γ, p, and q showed that the trajectories are nearly on top of one another when drawn on a graph plotter. Here, we used the above algorithm to convert the pressure values as the system of cracks grows, i.e. we introduced a hypothetical well of radius R. Changing over to such a problem makes it possible to obtain a system of linear equations of reasonable size for numerical calculations to describe the mutual effect of two systems of cracks located next to one another. In order to reduce the order of the system of equations to be solved, we will limit ourselves to

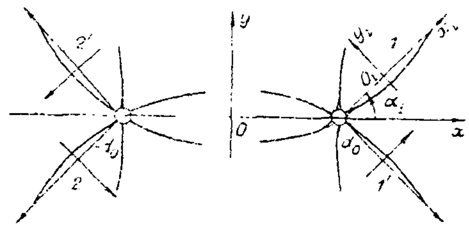

Fig. 7

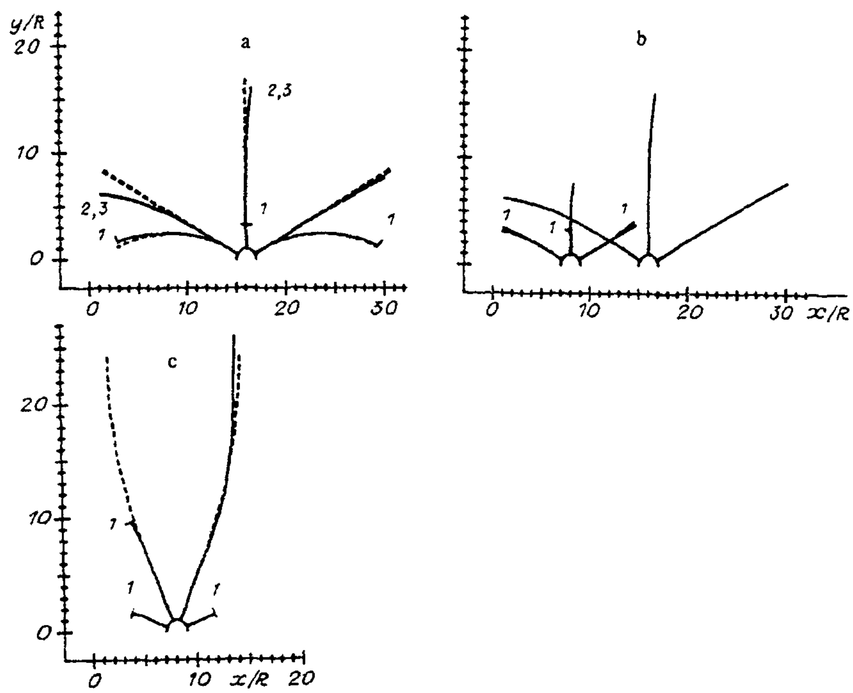

Fig. 8

consideration of the variant shown in Fig. 7. Here, we assume that N is an even number. In this case, $\alpha_1 = \pi/N$, $\gamma = \pi/2$. The problem is then symmetric relative to the axes Ox and Oy. Thus, if $T_{i1} = e^{i\alpha_1}\omega_i(\xi) + d_0 + x_i^0 + iy_i^0$ is the form of the i-th trajectory with subscript 1 (Fig. 1) and $g_i'(\xi)$ is the sought solution, then the following relations are valid

$$\begin{aligned} &T_{i1'} = \overline{T}_{i1},\ \overline{g_i'(\xi)},\ -\alpha_i \ (\text{line } 1'),\\ &T_{i2} = -\underline{T_{i1}},\ \underline{g_i'(\xi)},\ \pi + \alpha_i \ (\text{line } 2),\\ &T_{i2'} = -T_{i1},\ g_i'(\xi),\ \pi - \alpha_i \ (\text{line } 2'). \end{aligned}$$

Using these expressions, we again obtain a system of singular integral equations of the form (3). Here,

$$P_n(\eta) = -p^0(\eta) - p_+ + \rho_n p_- e^{2i\gamma},$$

$$\zeta_{kn}(\xi, \eta) = \zeta_{kn}(T_k, T_n) - \zeta_{kn}(d_0, T_n),$$

$$r_{kn}(\xi, \eta) = r_{kn}(T_k, T_n) - r_{kn}(d_0, T_n),$$

$$s_{kn}(\xi, \eta) = s_{kn}(T_k, T_n) - s_{kn}(d_0, T_n),$$

$$s_{kn}(T_k, T_n) = r_{kn}(\overline{T}_k, T_n),\ T_k(-1) = d_0,$$

$$\zeta_{kn}(T_k, T_n) = 2T_k\left(\frac{1}{T_k^2 - T_n^2} + \frac{1}{T_k^2 - \overline{T}_n^2}\right),\quad r_{kn}(T_k, T_n) = \frac{2\overline{T}_k}{T_k^2 - \overline{T}_n^2} - \frac{\overline{T}_k - T_n}{(T_k - \overline{T}_n)^2} - \frac{\overline{T}_k + T_n}{(T_k + \overline{T}_n)^2}.$$

The kernels of the integral equations have the property $R_{kn}(-1, \eta) = S_{kn}(-1, \eta) = 0$. As before, we take $\varphi_i(-1) = 0$, $i = 1, N/2$ as the auxiliary conditions. We use the relations presented above to recalculate the pressure acting in the cracks at each stage of propagation, i.e. we assume that they begin propagating from the boundary of a circular hole of radius R.

All subsequent calculations were performed for well charges 2 m long and $R = 0.0525$ under the condition that the gases penetrate the cracks. The criterion for ending the calculations was that the cracks not intersect, so the computation was stopped when the coordinate x of one of the cracks went below R.

The solid lines in Fig. 8a show the trajectories of cracks from the right charge when the left charge was placed the distance $2d_0 = 32R$ from it. The dashed lines show the trajectories for a similar system of cracks when there was no left charge. Trajectories 1 are an example of the interaction of "weak" charges. They were calculated until the cracks stopped, and they correspond to the initial values $r_0 = 0.018$ m, $p = -35$ MPa, and $q = -55$ MPa. Trajectories 2 and 3 correspond to the parameter values $p = -5$ MPa, $q = -6$ MPa, $r_0 = 0.02$-0.03 m and $p = -6$ MPa, $q = -5$ MPa, $r_0 = 0.03$ m. Here, the "weaker" external field $|\Delta p| = 1$ MPa, while in the first case $|\Delta p| = 20$ MPa. The trajectories nearly coincide for the chosen r_0. The cracks in variants 2 and 3 had velocities of the order v_0 at the moment the computation was completed, i.e. when the external field was sufficiently weak, the interaction of the charges became significant at $r_0 > 0.02$ m for the cracks closest to the nearby charge.

This is confirmed by the trajectories (Fig. 8b) which illustrate the effect of charge radius r_0 in a completely symmetric, uniform field of external stresses $p = q = -0.1$ MPa at $d_0/R = 8$ and 16. Trajectories 1 (the upper trajectories) correspond to $r_0 = 0.008$ m, while they nearly coincide at $r_0 = 0.013$-0.04 m. For $d_0/R = 16$, they coincide at $r_0 = 0.013$-0.018 m. The trajectories were straight in the absence of a left charge.

Figure 8a shows the effect of an adjacent charge when the uniform external field is uniaxial. Here, $N = 8$, $d_0/R = 8$, and $r_0 = 0.01$ m. In this case, trajectories 1 correspond to the case when the compressive field ($p = -5$ MPa, $q = 0$) is orthogonal to the line connecting the centers of the charges. The dashed lines represent the trajectories obtained in the absence of a left charge. Here, the first and fourth cracks travelled a somewhat greater distance before stopping than in the case of a single charge. The third crack was displaced toward the adjacent charge and stopped, while the second crack was travelling at 440 m/sec when the computation was ended. If the compressive stresses at infinity had been directed along the line connecting the charges ($p = 0$, $q = -5$ MPa), the effect of the adjacent charge in the given region would have been negligible. In this case, the second and third cracks would have undergone almost no displacement. Summing up the results, we note that if the external compressive field is parallel to the line connecting the charges, then the second charge will not have a significant effect on the trajectories of the cracks. If the external field is orthogonal to the same line, the effect will be manifest mainly on the cracks closest to the second charge.

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